***Note:***  *2 bonus marks, if the questions are solved in ascending order.*

***Relations and Functions* *(4 x 3 + 3 x 3 = 21)***

1. Determine, whether the relation *R* on the set of all people is reflexive, symmetric, and transitive where if and only if,
2. *x* is taller than *y*

Not reflexive: I am not taller than myself.

Not symmetric: X is taller than me, I am not taller than X.

Transitive.

1. *x* and *y* are born on same day

Reflexive, Symmetric, Transitive.

1. *x* has the same first name as *y*

Reflexive, Symmetric, Transitive.

1. *x* and *y* have same grandparents.

Relation *R* would be like:

Reflexive:

Symmetric: My and my brother's or cousin's grandparents can be same.

Not Transitive: It is not necessary.

1. Determine, whether the relation *R* on the set of all webpages is reflexive, symmetric, and transitive where if and only if,
2. Everyone who has visited webpage *a*, has also visited webpage *b*

Relation *R* could be

Reflexive, not-symmetric, transitive

1. There are no common links found on both webpage *a* and webpage *b*

Relation *R* could be

Not-Reflexive, Symmetric, not-transitive

1. There is a webpage that includes links to both webpage *a* and webpage *b*

Relation *R* could be

Not-Reflexive, Symmetric, not-transitive

***Loops and Time Complexity (4 x 6 = 24 Marks)***

1. Determine time complexity for the following loops. Draw table, and explicitly mention value of *k* and the range of *k* respectively.

***Step 1 (Table):***

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | 0 | 2 | 4 | 6 | … |  |

***Step 2 (value of k):***

***Step 3 (range of k):***

***Step 4 (cardinality of range (k)):***

***Step 5 (Total Complexity):***

***Step 1 (Table):***

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i |  |  |  | … |  |
| C(i) |  |  |  |  |  |

***Step 2 (value of k):***

***Step 3 (range of k):***

***Step 4 (cardinality of range (k)):***

***Step 5 (Total Complexity):***

***Counting (4 x 4 + 5 + 4 = 25 Marks)***

1. There are 18 mathematics major students and 325 computer science major students at a college.
   1. In how many ways can two students be picked so that one is a mathematics major and the other is a computer science major?

**Solution:** The product rule applies here, since we want to do two things, one after the other. First, since there are 18 mathematics majors, and we are to choose one of them, there are 18 ways to choose the mathematics major. Then we must choose the computer science major from among the 325 computer science majors, and that can clearly be done in 325 ways. Therefore, there are, by the product rule, 18 · 325 = 5850 ways to pick the two representatives.

* 1. In how many ways can one student be picked who is either a mathematics major or a computer science major?

**Solution:** The sum rule applies here, since we want to do one of two mutually exclusive things. Either we can choose a mathematics major to be the representative, or we can choose a computer science major. There are 18 ways to choose a mathematics major, and there are 325 ways to choose a computer science major. Since these two actions are mutually exclusive (no one is both a mathematics major and a computer science major), and since we want to do one of them or the other, there are 18 + 325 = 343 ways to pick the representative.

1. A palindrome is a string whose reversal is identical to the actual string. How many bit strings of length *n* are palindromes?

**Solution:** The trick here is to realize that a palindrome of length n is completely determined by its first bits. This is true because once these bits are specified, the remaining bits, read from right to left, must be identical to the first bits, read from left to right. Furthermore, these first bits can be specified at will, and by the product rule there are ways to do so.

1. A group contains *2n* men and 2*n* women. How many ways are there to arrange these people in a row if the men and women alternate?

**Solution:** We assume that the row has a distinguished head. Consider the order in which the men appear relative to each other. There are men, and all of the arrangements are allowed. Similarly, there are arrangements in which the women can appear. Now the men and women must alternate, and there are the same number of men and women; therefore, there are exactly two possibilities: either the row starts with a man and ends with a woman or else it starts with a woman and ends with a man . We have three tasks to perform, then: arrange the men among themselves, arrange the women among themselves, and decide which sex starts the row. By the product rule there are ways in which this can be done.

1. A club has 25 members.
   1. How many ways are there to choose four members of the club to serve on an executive committee?

**Solution:** Since the order of choosing the members is not relevant (the offices are not differentiated), we need to use a combination. The answer is clearly .

* 1. How many ways are there to choose a president, vice president, secretary, and treasurer of the club, where no person can hold more than one office?

**Solution:** Here we need a permutation, since the order matters (we choose first a president, then a vice president, then a secretary, then a treasurer). The answer is clearly .

1. A computer network consists of six computers. Each computer is directly connected to zero or more of the other computers. Show that there are at least two computers in the network that are directly connected to the same number of other computers.

**Solution:** Let ***c***, be the number of computers that the *ith* computer is connected to. Each of these integers is between *0* and *5*, inclusive. Let us therefore show that not all of the numbers can be used. The only way that the value *5* can appear as one of the *c(i)'s* is if one computer is connected to each of the others. In that case, the number *0* cannot appear, since no computer could be connected to none of the others. Thus, not both *5* and *0* can appear in our list, and the above argument is valid.

1. What is the coefficient of in the expansion of ?

**Solution:** Using the binomial theorem, we see that the term involving in the expansion of is . Therefore, the coefficient is .